ENERGY LESSON 7

Collisions

A **collision** occurs when two or more objects hit each other.

When objects collide, each object feels a force for a short amount of time. This force imparts an impulse, or changes the momentum of each of the colliding objects.

But if the system of particles is isolated, we know that momentum is conserved. Therefore, while the momentum of each individual particle involved in the collision changes, the total momentum of the system remains constant.

The procedure for analyzing a collision depends on whether the process is **elastic** or **inelastic**.

ELASTIC COLLISIONS

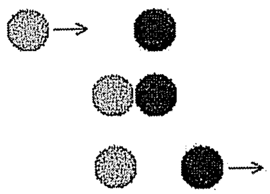
The total kinetic energy of all objects before the collision equals the total kinetic energy of all objects after the collision.

Notice that rather than just saying "energy is conserved" (which would imply that we need to take into account all kinds of energy), we have to focus on only ***kinetic energy***.

Unlike conservation of energy which refers to the changing forms of energy of an object, kinetic energy conservation refers to a system of conservation, one object transferring its kinetic energy to another object as kinetic energy.

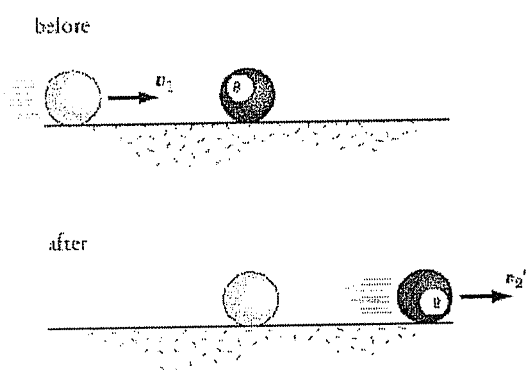
An elastic collision occurs when the two objects "bounce" apart when they collide.

EXAMPLE: Two identical rubber balls collide.



The first rubber ball hits the second and comes to a stop, while the second ball moves forward Almost no energy is lost to sound, heat, or deformation. The first rubber ball deforms, but then quickly bounces back to its former shape, and transfers almost all the kinetic energy to the second ball.

EXAMPLE: Watch a moving cue ball hit a resting pool ball (same mass). At impact, the cue ball stops, but transfers all of its kinetic energy to the other ball, resulting in the hit ball rolling with the initial speed of the cue ball.



Some kinetic energy is converted into sound energy when pool balls collide—otherwise, the collision would be silent—and a very small amount of kinetic energy is lost to friction. However, the **dissipated energy is such a small fraction** of the ball's kinetic energy that we can treat the collision as elastic or nearly elastic.

EXAMPLE: A perfectly elastic collision would be a super-bouncy ball. If you were to drop it, it would bounce all the way back up to the original height at which it was dropped. This would be considered a perfectly elastic collision as no kinetic energy is dissipated.

An *elastic collision* follows the Law of Conservation of Momentum, which states "the total amount of momentum before a collision is equal to the total amount of momentum after a collision."

(Recall, momentum is conserved in all collisions that occur in closed, isolated systems….where there are no external forces acting on the two objects, such as a forces from something outside the system like applied force, air resistance etc., internal force cannot change the total momentum of the system, such as friction between the balls.)

REAL WORLD: A car's bumper works by using this principle to prevent damage. In a low speed collision, the kinetic energy is small enough that the bumper can deform and then bounce back, transferring all the energy directly back into motion. Almost no energy is converted into heat, noise, or damage to the body of the car, as it would in an inelastic collision.

However, car bumpers are often made to collapse if the speed is high enough, and not use the benefits of an elastic collision. The rational is that if you are going to collide with something at a high speed, it is better to allow the kinetic energy to crumple the bumper in an inelastic collision than let the bumper shake you around as your car bounces in an elastic collision.

INELASTIC COLLISONS

In an *inelastic collision*, momentum is conserved, however, you'd measure that the kinetic energy after the collision is less than the kinetic energy before.

Most collisions are inelastic because kinetic energy is transferred or "lost" to other forms of energy during the collision process such as:

1.Friction between the objects could cause some of it to be converted to **heat** (thermal energy).

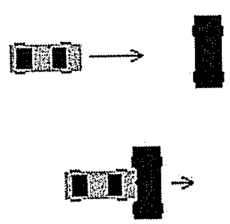
2.If the **object was permanently changed** (broken, bent, snapped, twisted, etc.) from its original shape. This includes if the objects are stuck together after the collision.

3.Some energy might have been converted into the energy of a **sound** or **light** that was released.

4. Some of the kinetic energy might have been converted into potential energy.

An inelastic collisions occurs when two objects collide and do not bounce away from each other.

EXAMPLE: Two cars colliding and moving together as one wreck.



EXAMPLE: When a rock shatters when hit by a bullet, the kinetic energy "lost" is great.

What about when two object stick together after a collision such as when a bullet or pie hit a wall?

The bullet or pie does not bounce at all and loses its momentum. All the kinetic energy goes into deforming the bullet or pie into a flat blob, making sound, heat, etc.

Even though rubber balls, pool balls (when hitting each other), and ping-pong balls may be assumed extremely elastic, there is still some bit of inelasticity in their collisions. If there were not, rubber balls would bounce forever.

The degree to which something is elastic or inelastic is usually found experimentally.

SUMMARY

Make sure that you keep these two types of collisions straight, based on whether or not kinetic energy is conserved.

Elastic collisions –Total kinetic energy before the collision equals total kinetic energy after. You can use conservation of kinetic energy ***with*** conservation of momentum .

Inelastic collisions –The kinetic energy changes. If the objects stick together after the collision, we say that the collision is completely or perfectly inelastic. Conservation of momentum still works in these collisions

QUESTIONS:

If you are asked to determine if a collision is elastic or inelastic:

Start off by calculating, individually, the kinetic energy of each object **before** the collision. Add them together to get the **total initial kinetic energy**.

Then calculate, individually, the kinetic energy of each object **after** the collision. Add them together to get the **total final kinetic energy**.

If the collision is ***elastic***, the two totals will be the same. If the collision is ***inelastic***, the initial total will be bigger than the final total.

QUESTION: Two billiard balls, assumed to have identical mass, collide in a perfectly elastic collision. Ball A is heading East at 12 m/s. Ball B is moving West at 8 m/s. Determine the post-collision velocities of Ball A and Ball B.

The - indicates West and the + indicates East)

Answer:

This collision is said to be perfectly elastic. Thus, both the total system momentum and the total system kinetic energy of the two objects is conserved. The momentum conservation equation can be written as

$$m_A \bullet v_{A\text{-before}} + m_B \bullet v_{B\text{-before}} = m_A \bullet v_{A\text{-after}} + m_B \bullet v_{B\text{-after}}$$

Since the balls are identical, their masses are the same. That is, $m_A = m_B = m$. The equation can be rewritten as:

$$m \bullet v_{A\text{-before}} + m \bullet v_{B\text{-before}} = m \bullet v_{A\text{-after}} + m \bullet v_{B\text{-after}}$$

Since each term of the equation contains the variable m, we can divide through by m and cancel m's from the equation. (If variable m is not the same for both objects, can't cancel them out….leave them in the equation.)

The equation can be rewritten as:

$$v_{A\text{-before}} + v_{B\text{-before}} = v_{A\text{-after}} + v_{B\text{-after}}$$ ←**Equation 1**

For elastic collisions, total system kinetic energy is conserved. The kinetic energy conservation equation is written as

$$0.5 \bullet m \bullet v_{A\text{-before}}^2 + 0.5 \bullet m \bullet v_{B\text{-before}}^2 = 0.5 \bullet m \bullet v_{A\text{-after}}^2 + 0.5 \bullet m \bullet v_{B\text{-after}}^2$$

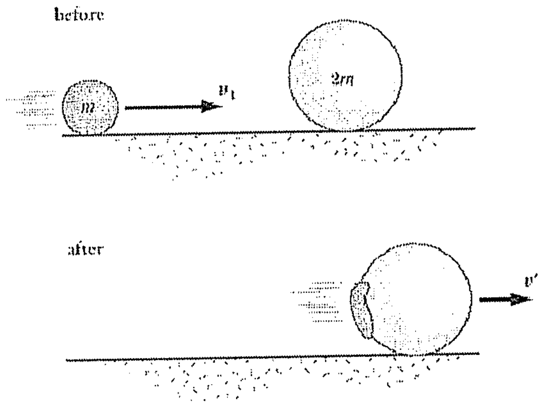
As shown in the book, this equation can be simplified to the form of ←**Equation 2**

$$v_{A\text{-before}} + v_{A\text{-after}} = v_{B\text{-before}} + v_{B\text{-after}}$$

The problem states the before-collision velocities of the two balls.

$$v_{A\text{-before}} = 12 \text{ cm/s (the + indicates east)}$$

EXAMPLE: Objects that stick together and move as one after a collision.



In a **perfectly inelastic** collision (as it is sometimes called), the two colliding objects stick together and move as a single unit after the collision. Such collisions are characterized by large losses in the kinetic energy of the system.

In the real world, there are no purely elastic or inelastic collisions, most collisions are a continuum between the two.

Type	**total kinetic energy**	**Comments**
perfectly inelastic	decreases to a minimum	objects stick together
Inelastic	decreases by any amount	all collisions between macroscopic bodies
partially elastic or nearly elastic	"nearly conserved"	billiard balls, bowling balls, steel bearings and other objects made from resilient materials
Elastic	absolutely conserved	collisions between atoms, molecules, subatomic particles and other similar microscopic bodies contrived collisions
superelastic	increases	between objects that release potential energy on contact, fictional superelastic materials like flubber

$$v_{B\text{-before}} = -8 \text{ cm/s (the - indicates west)}$$

These two values can be substituted into equations 1 and 2 above.

$$12 \text{ cm/s} - 8 \text{ cm/s} = v_{A\text{-after}} + v_{B\text{-after}} \quad \longleftarrow \textbf{Equation 3}$$

$$12 \text{ cm/s} + v_{A\text{-after}} = -8 \text{ cm/s} + v_{B\text{-after}} \quad \longleftarrow \textbf{Equation 4}$$

Now the problem has been reduced to two equations and two unknowns. Such a problem can be solved in numerous ways. One method involved using Equation 3 to develop an expression for $v_{A\text{-after}}$ in terms of $v_{B\text{-after}}$. This expression for $v_{A\text{-after}}$ can then be substituted into Equation 4. The value of $v_{B\text{-after}}$ can then be determined. This is shown below.

From Equation 3:

$$v_{A\text{-after}} = 12 \text{ cm/s} - 8 \text{ cm/s} - v_{B\text{-after}}$$

$$v_{A\text{-after}} = 4 \text{ cm/s} - v_{B\text{-after}} \quad \longleftarrow \textbf{Equation 5}$$

This expression for $v_{A\text{-after}}$ in terms of $v_{B\text{-after}}$ can now be substituted into equation 4. This is shown below. The subsequent algebraic manipulation is shown as well.

$$12 \text{ cm/s} + 4 \text{ cm/s} - v_{B\text{-after}} = -8 \text{ cm/s} + v_{B\text{-after}}$$

$$12 \text{ cm/s} + 4 \text{ cm/s} + 8 \text{ cm/s} - v_{B\text{-after}} = + v_{B\text{-after}}$$

$$24 \text{ cm/s} = + v_{B\text{-after}} + v_{B\text{-after}}$$

$$24 \text{ cm/s} = 2\, v_{B\text{-after}}$$

$$\mathbf{v_{B\text{-after}} = +12 \text{ cm/s}}$$

Now that the value of $v_{B\text{-after}}$ has been determined, it can be substituted into the original expression for $v_{A\text{-after}}$ (Equation 5) in order to determine the numerical value of $v_{A\text{-after}}$. This is shown below.

$$v_{A\text{-after}} = 4 \text{ cm/s} - v_{B\text{-after}}$$

$$v_{A\text{-after}} = 4 \text{ cm/s} - 12 \text{ cm/s}$$

$$\mathbf{v_{A\text{-after}} = -8 \text{ cm/s}}$$

Let's figure out a question and then see if it is elastic or inelastic.

QUESTION 3: One way to test the speed of a bullet shot from a gun is to use a device called a ballistic pendulum. Because it is based on well understood physics, it can give very accurate results even though the equipment is quite simple. A block of material such as wood is hung from supporting wires as shown below. When the bullet is shot at the pendulum, it hits and becomes embedded in the pendulum. Together, the pendulum and the bullet swing upwards. By measuring the maximum height that the pendulum and bullet swing to, the speed of the bullet just before impact can be calculated. For this problem, a 0.0200 kg bullet collides with a 5.7500 kg pendulum. After the collision, the pair swings up to a maximum height of 0.386 m . **Determine** the velocity of the bullet just before impact.

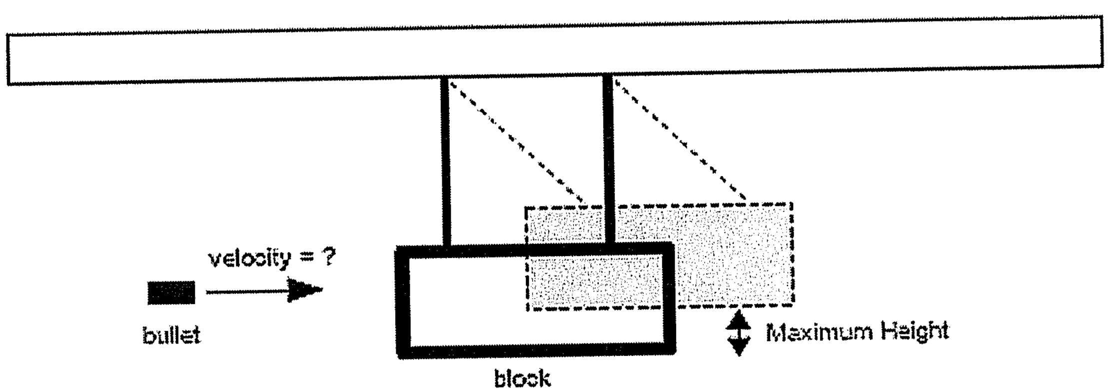


Illustration 1: A Ballistic Pendulum

1. First, we need to decide if this is an elastic or inelastic collision. Figuring that out from the start will give us an idea of how we need to go after it.

Since a bullet hitting a block causes deformation of the block and the bullet, we can assume the collision is **inelastic**. That means that we will ***not*** be able to say that kinetic energy is conserved during the collision itself. We can still use regular conservation of energy for the whole thing swinging like a pendulum. We will be able to use conservation of momentum for the collision of the pendulum and the bullet, since momentum always works.

2. Conservation of Energy

We can use the information about the whole pendulum-bullet swinging upwards like a pendulum to figure out some stuff that is happening *after* the collision. We know that the kinetic energy of pendulum-bullet just after the collision is turned into gravitational potential energy as it swings upwards, so...

$$KE_i + PE_i + W_{ext} = KE_f + PE_f$$

(assume that this is an isolated system and does not have external forces such as air resistance, etc)

$$\frac{1}{2}(m + M)v^2 = (m + M)gh$$

$$\frac{1}{2} v = gh$$

$$v = 2gh$$

$$v = 2*9.81*0.386$$

$$v = 2.75 m/s$$

This is the velocity of the pendulum-bullet just *after* the collision has happened.

3. Conservation of Momentum

Now we have enough information to calculate the conservation of momentum before and after the collision, which will allow us to calculate the velocity of the bullet just before it hit the pendulum.

$$p_{total} = p_{total}'$$

$$m_p v_p + m_b v_b = m_p v_p' + m_b v_b'$$

$$0 + m_b v_b = v'(m_p + m_b)$$

$$0.0200\ v_b = 2.75\ (5.7500 + 0.0200)$$

$$0.0200\ v_b = 2.75\ (5.7700)$$

$$0.0200\ v_b = 15.9$$

$$v_b = 794\ m/s$$

The bullet was traveling at 794 m/s just before it hit the pendulum.

QUESTION 3: In example 2 we *assumed* that the collision was inelastic (it's macroscopic and we ***didn't*** use conservation of kinetic energy.) Using the information from example 1, **determine** if the collision was elastic or inelastic.

Answer:

Initially, only the bullet was moving. We only need to calculate its kinetic energy and use that value as the **total initial kinetic energy**.

$KE_i = (1/2)mv^2 = 1/2*0.0200*(794)^2 = 6304.36\ J = 6.30 \times 10^3 J$

Just after the collision, the bullet and the block move together as one mass at the same velocity. We'll only need to do one calculation for the **total final kinetic energy**.

$KE_f = (1/2)(m+M)v^2 = 1/2*5.7700*(2.75)^2 = 21.8178125\ J = 21.8\ J$

It's obvious that after the collision there is considerably less kinetic energy than at the start. This is an **inelastic** collision.

$\Delta KE = KE_f - KE_i$

$\Delta KE = 21.8178 - 6304.36 = -6.28 \times 10^3$ J This is the amount of kinetic energy lost to other forms of energy when the bullet collided with the wood block.

Fraction lost is= $\Delta KE/KE_i = -6282.54/6304.36 = -99.7\%$ (99.7 % lost) Energy went into potential energy, deformation, heat, sound, etc.

In fact, only about 0.346% of the kinetic energy remained after the collision. To get the percentage, just divide the **final** by the **initial.**

ANIMATION- http://www.youtube.com/watch?v=SiAil1ljN9A (ELASTIC COLLISION)

ON LINE EXAMPLE OF INELASTIC AND ELASTIC COLLISIONS :

The web site:
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/AirTrack/AirTrack.html

ENERGY LESSON 7 HOMEWORK

DO WORKSHEET (ENERGY IN COLLISIONS)
Comment on the outcomes, paying strict attention to the energy and momentum of each system before and after the collision. **Does the outcome describe a completely inelastic, partially inelastic, partially elastic, completely elastic, or impossible collision?** Provide a brief explanation to accompany each answer.

Energy in Collisions

Type	total kinetic energy	Comments
perfectly inelastic	decreases to a minimum	objects stick together
Inelastic	decreases by any amount	all collisions between macroscopic bodies
partially elastic or nearly elastic	"nearly conserved"	billiard balls, bowling balls, steel bearings and other objects made from resilient materials
Elastic	absolutely conserved	collisions between atoms, molecules, subatomic particles and other similar microscopic bodies
superelastic	increases	contrived collisions between objects that release potential energy on contact, fictional superelastic materials like flubber

ANSWER KEY for worksheet

Solutions …

1. Here we are given the initial conditions of the two colliding objects.

$$\sum p = m_1v_1 + m_2v_2 = (8\text{ kg})(+6\text{ m/s}) + (4\text{ kg})(-9\text{ m/s}) = +12\text{ kg·m/s}$$

$$\sum K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(8\text{ kg})(6\text{ m/s})^2 + \frac{1}{2}(4\text{ kg})(9\text{ m/s})^2 = 306\text{ J}$$

Momentum is a vector quantity, so the total momentum is found by a vector sum. Since the momentums of the two objects are in opposite directions one of them is going to be negative. Since positive answers are preferred over negative ones, let's choose right as the positive direction. This gives us a total momentum of +12 kg·m/s. Energy being a scalar is much easier to handle, especially here since the only energy that matters is kinetic (which is always positive). The total mechanical energy of this system in 306 J.

1. The first outcome we'll be examining has the two objects sticking together and moving off to the right.

$$\sum p' = (m_1 + m_2)v' = (8 + 4 \text{ kg})(+1 \text{ m/s}) = +12 \text{ kg·m/s}$$

$$\sum K' = \frac{1}{2}(m_1 + m_2)v'^2 = \frac{1}{2}(8 + 4 \text{ kg})(1 \text{ m/s})^2 = 6 \text{ J}$$

This is a sensible outcome since the initial total momentum is positive (to the right). Calculations show that the final total momentum is still +12 kg·m/s, but the final total energy has dropped significantly to a relatively low value of 6 J. Momentum was conserved, but mechanical energy was lost. This a classic example of an <u>inelastic collision</u>. (Some would even call this a <u>completely inelastic collision</u>.) The lost energy has likely gone into plastic deformation of the two objects (given the distorted edges shown in the diagram).

2. Here the two objects have separated after collision and are moving in opposite directions. Each is moving more slowly than it was before the collision. This hints at a loss of mechanical energy.

$$\sum p' = m_1v_1' + m_2v_2' = (8 \text{ kg})(-1 \text{ m/s}) + (4 \text{ kg})(+5 \text{ m/s}) = +12 \text{ kg·m/s}$$

$$\sum K' = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = \frac{1}{2}(8 \text{ kg})(1 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(+5 \text{ m/s})^2 = 54 \text{ J}$$

Momentum was conserved as it should be, but mechanical energy was lost making this an <u>inelastic collision</u>. Since more energy was retained than in the previous outcome, some would call this a <u>partially inelastic collision</u>. Lost energy is not a big deal and does not violate the conservation of energy. The energy wasn't destroyed in this outcome. It just turned into a form that isn't easy to see with our eyes -- internal energy. When kinetic energy transforms into internal energy, either the temperature of the system increases or it experience a phase change (melting, for example). <u>Internal energy</u> will be dealt with in more detail later in this book.

3. This outcome is similar to the previous one only now the objects are moving a bit more quickly. Still, their speeds after the collision are slower than before.

$$\sum p' = m_1 v_1' + m_2 v_2' = (8\text{ kg})(-3\text{ m/s}) + (4\text{ kg})(+9\text{ m/s}) = +12\text{ kg·m/s}$$

$$\sum K' = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 = \frac{1}{2}(8\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(4\text{ kg})(9\text{ m/s})^2 = 198\text{ J}$$

Momentum was conserved and energy was lost, but to a lesser extent than in the previous outcome. Of all the outcomes so far, this inelastic collision is the least inelastic. Whether one would call it partially inelastic or partially elastic doesn't really matter.

4. Here we see an elastic collision. (Some would even call this a perfectly elastic collision.)

$$\sum p' = m_1 v_1' + m_2 v_2' = (8\text{ kg})(-4\text{ m/s}) + (4\text{ kg})(+11\text{ m/s}) = +12\text{ kg·m/s}$$

$$\sum K' = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 = \frac{1}{2}(8\text{ kg})(4\text{ m/s})^2 + \frac{1}{2}(4\text{ kg})(11\text{ m/s})^2 = 306\text{ J}$$

Momentum and total mechanical energy of the system were both conserved. The total kinetic energy of the two objects after the collision is the same as it was before. In the macroscopic world such an outcome would never happen. The results above show the limit of what could happen. That is, macroscopic objects will always have less total mechanical energy after a collision than before. Never equal to or greater than.

5. This outcome is difficult to explain.

$$\sum p' = m_1 v_1' + m_2 v_2' = (8\text{ kg})(-6\text{ m/s}) + (4\text{ kg})(+15\text{ m/s}) = +12\text{ kg·m/s}$$

$$\sum K' = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 = \frac{1}{2}(8\text{ kg})(6\text{ m/s})^2 + \frac{1}{2}(4\text{ kg})(15\text{ m/s})^2 = 594\text{ J}$$

Momentum was conserved, but mechanical energy increased. How could this happen? Where did the extra energy come from? Since this is a problem about generic objects, we are free to contrive all sorts of explanations. Perhaps there was a compressed spring on one of the objects, or a chemical explosive, or the two objects were small mammals that kicked off of each other after they collided. I think "contrive" was an appropriate word choice on my part to describe what we're doing here. This outcome seems improbable as it violates the conservation of mechanical energy in a way different from the previous outcomes.

6. This outcome is also perplexing.

$$\sum p' = m_1v_1' + m_2v_2' = (8\text{ kg})(-8\text{ m/s}) + (4\text{ kg})(+5\text{ m/s}) = -44\text{ kg·m/s}$$

$$\sum K' = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = \frac{1}{2}(8\text{ kg})(8\text{ m/s})^2 + \frac{1}{2}(4\text{ kg})(5\text{ m/s})^2 = 306\text{ J}$$

Impossible, unless its Flubber!

1. Which of the following statements are true about elastic and inelastic collisions?

   a. Perfectly elastic and perfectly inelastic collisions are the two opposite extremes along a continuum; where a particular collision lies along the continuum is dependent upon the amount kinetic energy which is conserved by the two objects.
   b. Most collisions tend to be partially to completely elastic.
   c. Momentum is conserved in an elastic collision but not in an inelastic collision.
   d. The kinetic energy of an object remains constant during an elastic collision.
   e. Elastic collisions occur when the collision force is a non-contact force.
   f. Most collisions are not inelastic because the collision forces cause energy of motion to be transformed into sound, light and thermal energy (to name a few).
   g. A ball is dropped from rest and collides with the ground. The higher that the ball rises upon collision with the ground, the more elastic that the collision is.
   h. A moving air track glider collides with a second stationary glider of identical mass. The first glider loses all of its kinetic energy during the collision as the second glider is set in motion with the same original speed as the first glider. Since the first glider lost all of its kinetic energy, this is a perfectly inelastic collision.
   i. The collision between a tennis ball and a tennis racket tends to be more elastic in nature than a collision between a halfback and linebacker in football.

Answer: AEFGI

a. **TRUE** - A perfectly elastic collision is a collision in which the total kinetic energy of the system of colliding objects is conserved. Such collisions are typically characterized by bouncing or repelling from a distance. In a perfectly inelastic collision (as it is sometimes called), the two colliding objects stick together and move as a single unit after the collision. Such collisions are characterized by large losses in the kinetic energy of the system.

b. **FALSE** - Few collisions are completely elastic. A completely elastic collision occurs only when the collision force is a non-contact force. Most collisions are either perfectly inelastic or partially inelastic.

c. **FALSE** - Momentum can be conserved in both elastic and inelastic collisions provided that the system of colliding objects is isolated from the influence of net external forces. It is kinetic energy that is conserved in a perfectly elastic collision.

d. **FALSE** - In a perfectly elastic collision, in an individual object may gain or lose kinetic energy. It is the system of colliding objects which conserves kinetic energy.

e. **TRUE** - Kinetic energy is lost from a system of colliding objects because the collision transforms kinetic energy into other forms of energy - sound, heat and light energy. When the colliding objects don't really collide in the usual sense (that is when the collision force is a non-contact force), the system of colliding objects does not lose its kinetic energy. Sound is only produced when atoms of one object make contact with atoms of another object. And objects only warm up (converting mechanical energy into thermal energy) when their surfaces meet and atoms at those surfaces are set into vibrational motion or some kind of motion.

f. **TRUE** - See above statement.

g. **TRUE** - If large amounts of kinetic energy are conserved when a ball collides with the ground, then the post-collision velocity is high compared to the pre-collision velocity. The ball will thus rise to a height which is nearer to its initial height.

h. **FALSE** - This is a perfectly elastic collision. Before the collision, all the kinetic energy is in the first glider. After the collision, the first glider has no kinetic energy; yet the second glider has the same mass and velocity as the first glider. As such, the second glider has the kinetic energy which the first glider once had.

i. **TRUE** - There is significant bounce in the collision between a tennis racket and tennis ball. There is typically little bounce in the collision between a halfback and a linebacker (though there are certainly exceptions to this one). Thus, the ball-racket collision tends to be more elastic.

2. A cricket ball, mass 0.500 kg, was bowled at 50.0 m/s at a batsman who misreads the ball and the 5.00 kg bat is knocked out of his hands, the ball rebounds at 25.0 m/s.

a) What is the initial momentum of the ball?

b) What is the momentum of the ball just after it hit the bat?

c) How much momentum is transferred to the bat?

d) Calculate the velocity of the bat as it leaves the batsman.

e) Calculate the kinetic energies before and after the collision of bat and ball. Is the collision elastic or inelastic?

Answer: 25 kg m/s, -12.5 kg m/s, 37.5 kg m/s, 7.5 m/s, inelastic

Answer:

2. a) Initial momentum of the ball = 0.5 x 50 = 25 kg m/s

b) Momentum of the ball just after it hit the bat = 0.5 x (-25) = -12.5 kg m/s (Taking positive velocity to the right.)

c) Momentum transferred to the bat =25 - (-12.5) = 37.5 kg m/s

d) Momentum of bat = 37.5 kg m/s so velocity of bat = (37.5)/5 = 7.5 m/s

e) Calculate the kinetic energies before and after the collision of bat and ball. Is the collision elastic or inelastic?

$KE = \frac{1}{2}mv^2$ so before collision $KE = \frac{1}{2}0.5(50)^2$ and KE = 625 J

After the collision KE of ball is $KE = \frac{1}{2}0.5(25)^2$ so KE = 156.25 J = 156 J

After collision Bat KE = $KE = \frac{1}{2}5(7.5)^2$ so KE = 140.63 J = 141 J

Total KE after collision is 156.25 + 140.63 =296.9 J = 297 J, so the collision is inelastic as KE before > KE after the collision

3. A ball of mass 0.20 kg is dropped from a height of 3.2 m onto a flat surface which it hits at 8.0 m/s. It rebounds to 1.8 m. ($g = 9.8$ m s^{-2})

a) What is the rebound speed just after impact?

b) What is the change in energy of the ball?

c) What momentum change has the ball between just touching the surface and leaving it?

Answer: -5.9 m/s, 6.4 J, 2.8 kg m/s

3\. a) This is an energy reminder $v_f^2 = v_i^2 + 2ad \rightarrow v = \sqrt{2gh}$ so $v = \sqrt{2x(9.8)x(1.8)}$ and speed = 5.9 m/s so velocity = - 5.9 m/s

b) $KE = \frac{1}{2}mv^2$ so before impact KE = 0.5 x 0.20 x 8.0 x 8.0 = 6.4 J

after impact KE = 0.5 x 0.20 x (-5.9) x (-5.9) = 3.5 J so KE change is $KE_f - KE_i$ = 3.5 - 6.4 = - 2.9J The ball loses 2.9 J of energy.

c) Momentum before collision = 0.2 x 8.0 = 1.6 kg m/s

Momentum of ball after = 0.2 x (-5.9) = -1.2 kg m/s

Change in momentum = 1.6 - (-1.2) = 2.8 kg m/s

4\. At an amusement park, twin brothers Tubby (m = 50 kg) and Chubby (m = 62 kg) occupy separate 36-kg bumper cars. Tubby gets his car cruising at 3.6 m/s and collides head-on with Chubby who is moving the opposite direction at 1.6 m/s. After the collision, Tubby bounces backwards at 0.5 m/s. Assuming an isolated system, determine ...

a. ... Chubby's post-collision speed.

b. ... the percentage of original kinetic energy which is lost as the result of the collision

Answer: (a) v = ~2.0 m/s (b) % KE Loss = ~70%

(a) Expressions for the total momentum of the system before and after the collision can be written. For the before-collision expression, Tubby is assigned a positive velocity value and Chubby is assigned a negative velocity value (since he is moving in the opposite direction). Furthermore, the mass of the bumper car must be figured into the total mass of the individually moving objects.

$$p_{total\text{-}before} = p_{Tubby\text{-}before} + p_{Chubby\text{-}before}$$

$$p_{total\text{-}before} = m_{Tubby} \bullet v_{Tubby\text{-}before} + m_{Chubby} \bullet v_{Chubby\text{-}before}$$

$$p_{total\text{-}before} = (86\ kg) \bullet (3.6\ m/s) + (98\ kg) \bullet (-1.6\ m/s)$$

For the before-collision expression, Tubby is assigned a negative velocity value (since he has bounced backwards in the opposite direction of his original motion. Chubby is assigned a velocity of **v** since his velocity is not known.

$$p_{total\text{-}after} = p_{Tubby\text{-}after} + p_{Chubby\text{-}after}$$

$$p_{total\text{-}after} = m_{Tubby} \bullet v_{Tubby\text{-}after} + m_{Chubby} \bullet v_{Chubby\text{-}after}$$

$$p_{total\text{-}after} = (86\text{ kg}) \bullet (-0.5\text{ m/s}) + (98\text{ kg}) \bullet (v_{Chubby\text{-}after})$$

$$p_{total\text{-}after} = (86\text{ kg}) \bullet (-0.5\text{ m/s}) + (98\text{ kg}) \bullet \mathbf{v}$$

Since the system is assumed to be isolated, the before-collision momentum expression can be set equal to the after-collision momentum expression. The equation can then be algebraically manipulated to solve for the post-collision velocity of Chubby.

$$(86\text{ kg}) \bullet (3.6\text{ m/s}) + (98\text{ kg}) \bullet (-1.6\text{ m/s}) = (86\text{ kg}) \bullet (-0.5\text{ m/s}) + (98\text{ kg}) \bullet \mathbf{v}$$

$$309.6\text{ kg}\bullet\text{m/s} - 156.8\text{ kg}\bullet\text{m/s} = (86\text{ kg}) \bullet (-0.5\text{ m/s}) + (98\text{ kg}) \bullet \mathbf{v}$$

$$152.8\text{ kg}\bullet\text{m/s} = -43\text{ kg}\bullet\text{m/s} + (98\text{ kg}) \bullet \mathbf{v}$$

$$152.8\text{ kg}\bullet\text{m/s} + 43\text{ kg}\bullet\text{m/s} = (98\text{ kg}) \bullet \mathbf{v}$$

$$195.8\text{ kg}\bullet\text{m/s} = (98\text{ kg}) \bullet \mathbf{v}$$

$$\mathbf{v} = (195.8\text{ kg}\bullet\text{m/s}) / (98\text{ kg})$$

$$\mathbf{v = 1.998\text{ m/s} = \sim 2.0\text{ m/s}}$$

(b) This collision is neither perfectly elastic (since the collision force is a contact force) nor perfectly inelastic (since the objects do not *stick together*). It is a partially elastic/inelastic collision. Since the collision is not perfectly elastic, there is a loss of total system kinetic energy during the collision. The before-collision and after-collision kinetic energy values can be calculated and the percentage of total KE lost can be determined.

The before-collision KE is based on before-collision speeds:

$$KE_{system\text{-}before} = KE_{Tubby\text{-}before} + KE_{Chubby\text{-}before}$$

$$KE_{system\text{-}before} = 0.5 \bullet m_{Tubby} \bullet v_{Tubby\text{-}before}^2 + 0.5 \bullet m_{Chubby} \bullet v_{Chubby\text{-}before}^2$$

$$KE_{system\text{-}before} = 0.5 \bullet (86\text{ kg}) \bullet (3.6\text{ m/s})^2 + 0.5 \bullet (98\text{ kg}) \bullet (1.6\text{ m/s})^2$$

$$KE_{system\text{-}before} = 557.28\text{ J} + 125.44\text{ J}$$

$$KE_{system\text{-}before} = 682.72\text{ J}$$

The after-collision KE is based on after -collision speeds:

$$KE_{system\text{-}after} = KE_{Tubby\text{-}after} + KE_{Chubby\text{-}after}$$

$$KE_{system\text{-}after} = 0.5 \bullet m_{Tubby} \bullet v_{Tubby\text{-}after} + 0.5 \bullet m_{Chubby} \bullet v_{Chubby\text{-}after}$$

5. In a physics lab, two carts collide elastically on a level, low-friction track. Cart A has a mass of 1.500 kg and is moving east at 36.5 cm/s. Cart B has a mass of 0.500 kg and is moving West at 42.8 cm/s. Determine the post-collision velocities of the two carts.

Answer: $v_{A\text{-}after}$ = -3.15 cm/s; $v_{B\text{-}after}$ = 76.15 cm/s

This is a perfectly elastic collision in which both momentum and kinetic energy are conserved. The method for solving this problem will be very similar to that used in Problem #68 above. Two equations will be developed using the momentum conservation and kinetic energy conservation principles. One equation will be used to develop an expression for v_A in terms of v_B. This expression will then be substituted into the second equation in order to solve for v_B. The original v_A expression can then be used to determine the v_A value. The solution is shown below.

The momentum conservation equation can be written as

$$m_A \bullet v_{A\text{-}before} + m_B \bullet v_{B\text{-}before} = m_A \bullet v_{A\text{-}after} + m_B \bullet v_{B\text{-}after}$$

$$(1.500\text{ kg}) \bullet (+36.5\text{ cm/s}) + (0.500\text{ kg}) \bullet (-42.8\text{ cm/s}) = (1.500\text{ kg}) \bullet v_{A\text{-}after} + (0.500\text{ kg}) \bullet v_{B\text{-}after}$$

$$33.35\text{ kg} \bullet \text{cm/s} = (1.500\text{ kg}) \bullet v_{A\text{-}after} + (0.500\text{ kg}) \bullet v_{B\text{-}after}$$ ← **Equation 1**

For elastic collisions, total system kinetic energy is conserved. The kinetic energy conservation equation is written as

$$0.5 \bullet m \bullet v_{A\text{-}before}^2 + 0.5 \bullet m \bullet v_{B\text{-}before}^2 = 0.5 \bullet m \bullet v_{A\text{-}after}^2 + 0.5 \bullet m \bullet v_{B\text{-}after}^2$$

As shown in the book, this equation can be simplified to the form of

$$v_{A\text{-}before} + v_{A\text{-}after} = v_{B\text{-}before} + v_{B\text{-}after}$$

$$36.5\text{ cm/s} + v_{A\text{-}after} = -42.8\text{ cm/s} + v_{B\text{-}after}$$

$$v_{A\text{-}after} = v_{B\text{-}after} - 79.3\text{ cm/s}$$ ← **Equation 2**

Now the problem has been reduced to two equations and two unknowns. Such a problem can be solved in numerous ways. Note that equation 2 represents an expression for $v_{A\text{-}after}$ in terms of $v_{B\text{-}after}$. This expression for $v_{A\text{-}after}$ can then be substituted into Equation 1. The value of $v_{B\text{-}after}$ can then be determined. This work is shown below. (To simplify the mathematics, the units will be dropped from the numerical values stated in the solution. When $v_{B\text{-after}}$ is solved for, its units will be in cm/s - the same units used for velocity in the above portion of the solution.)

$$33.35 = (1.500) \bullet (v_{B\text{-after}} - 79.3) + (0.500) \bullet v_{B\text{-after}}$$

$$33.35 = 1.500 \bullet v_{B\text{-after}} - 118.95 + 0.500 \bullet v_{B\text{-after}}$$

$$152.30 = 2.0 \bullet v_{B\text{-after}}$$

$$\mathbf{v_{B\text{-after}} = 76.15\ cm/s}$$

Now that the value of $v_{B\text{-after}}$ has been determined, it can be substituted into the original expression for $v_{A\text{-after}}$ (Equation 2) in order to determine the numerical value of $v_{A\text{-after}}$. This is shown below.

$$v_{A\text{-after}} = v_{B\text{-after}} - 79.3\ \text{cm/s}$$

$$v_{A\text{-after}} = 76.15\ \text{cm/s} - 79.3\ \text{cm/s}$$

$\mathbf{v_{A\text{-after}} = -3.15\ cm/s}$

6. A classic physics demonstration involves firing a bullet into a block of wood suspended by strings from the ceiling. The height to which the wood rises below its lowest position is mathematically related to the pre-collision speed of the bullet. If a 9.7-gram bullet is fired into the center of a 1.1-kg block of wood and it rises upward a distance of 33 cm, then what was the pre-collision speed of the bullet?

Answer: 2.9 x 10^2 m/s

Here is another instance in which momentum principles must be combined with content learned in other units in order to complete an analysis of a physical situation. The collision involves the inelastic collision between a block of wood and bullet. The bulled lodges into the wood and the two objects move with identical velocity after the collision. The kinetic energy of the wood and bullet is then converted to potential energy as the combination of two objects rises to a final resting position.

Energy conservation can be used to determine the velocity of the wood-bullet combination immediately after the collision. The kinetic energy of the wood-bullet combination is set equal to the final potential energy of the wood-bullet combination and the equation is manipulated to solve for the post-collision velocity of the wood-bullet combination. The work is shown here:

$$0.5 \bullet (m_{wood} + m_{bullet}) \bullet v_{combination\text{-after}}^2 = (m_{wood} + m_{bullet}) \bullet (9.8\ \text{m/s}^2) \bullet (0.33\ \text{m})$$

$$0.5 \cdot v_{combination\text{-}after}^2 = (9.8\ m/s^2) \cdot (0.33\ m)$$

$$v_{combination\text{-}after}^2 = 2 \cdot (9.8\ m/s^2) \cdot (0.33\ m)$$

$$v_{combination\text{-}after}^2 = 6.468\ m^2/s^2$$

$$v_{combination\text{-}after} = 2.5432\ m/s$$

Now momentum conservation can be used to determine the pre-collision velocity of the bullet ($v_{bullet\text{-}before}$). The known information is:

$m_{wood} = 1.1$ kg; $m_{bullet} = 9.7$ g $= 0.0097$ kg; $v_{wood\text{-}after} = 2.5432$ m/s; $v_{bullet\text{-}after} = 2.5432$ m/s

Expressions for the total system momentum can be written for the before- and after-collision situations.

Before Collision: $p_{total\text{-}before} = (1.1\ kg) \cdot (0\ m/s) + (0.0097\ kg) \cdot (v_{bullet\text{-}before})$

After Collision: $p_{total\text{-}after} = (1.1\ kg) \cdot (2.5432\ m/s) + (0.0097\ kg) \cdot (2.5432\ m/s)$

Assuming momentum conservation, these expressions are set equal to each other and then algebraically manipulated to solve for the unknown (m_B).

$$(1.1\ kg) \cdot (0\ m/s) + (0.0097\ kg) \cdot (v_{bullet\text{-}before}) = (1.1\ kg) \cdot (2.5432\ m/s) + (0.0097\ kg) \cdot (2.5432\ m/s)$$

$$(0.0097\ kg) \cdot (v_{bullet\text{-}before}) = (1.1\ kg) \cdot (2.5432\ m/s) + (0.0097\ kg) \cdot (2.5432\ m/s)$$

$$(0.0097\ kg) \cdot (v_{bullet\text{-}before}) = 2.8222\ kg \cdot m/s$$

$$v_{bullet\text{-}before} = (2.8222\ kg \cdot m/s) / (0.0097\ kg)$$

$$v_{bullet\text{-}before} = 290.95\ m/s$$

$\mathbf{v_{bullet\text{-}before} = \sim 2.9 \times 10^2\ m/s}$

7. CHALLEGE QUESTION: You have all the tools to figure this question out. Use what you know and figure out the puzzle.

A 1.72-kg block of soft wood is suspended by two strings from the ceiling. The wood is free to rotate in pendulum-like fashion when a force is exerted upon it. A 8.50-g bullet is fired into the wood. The bullet enters the wood at 431 m/s and exits the opposite side immediately thereafter (i.e. it has exerted a force on the wood giving it a velocity over an instant). If the wood rises to a height of 13.8 cm, then what is the exit speed of the bullet?

Answer: $\mathbf{v_{bullet\text{-}after} = \sim 98\ m/s}$

The difficulty of this problem lies in the fact that information from other units (work and energy) must be combined with the momentum information from this unit to arrive at a solution to the problem. In this scenario there is a collision between a stationary block of wood and a moving bullet. The impulse causes the block of wood to be set into motion and the bullet to slow down. Momentum can be assumed to be conserved. Once set into motion, the block of wood rises in pendulum-like fashion to a given height. Its energy of motion (kinetic energy) is transformed into energy of vertical position (potential energy). The post-collision speed of the wood can be determined using energy conservation equations.

To begin the solution, the final height of the wood is used to determine the post-collision speed of the wood.

$$KE_{initial} = PE_{final}$$

$$0.5 \bullet \cancel{m_{wood}} \bullet v_{wood}^2 = \cancel{m_{wood}} \bullet g \bullet h_{wood}$$

$$v_{wood}^2 = 2 \bullet g \bullet h_{wood}$$

$$v_{wood} = SQRT(2 \bullet g \bullet h_{wood})$$

$$v_{wood} = SQRT[2 \bullet (9.8\ m/s^2) \bullet (0.138\ m)]$$

$$v_{wood} = SQRT[2.7048\ m^2/s^2]$$

$$v_{wood} = 1.6446\ m/s$$

Immediately following the emergence of the bullet from the wood, the wood block is moving with a speed of 1.6446 m/s. Knowing this, momentum conservation can be applied to determine the post-collision speed of the bullet.

$$m_{wood} \bullet v_{wood\text{-}before} + m_{bullet} \bullet v_{bullet\text{-}before} = m_{wood} \bullet v_{wood\text{-}after} + m_{bullet} \bullet v_{bullet\text{-}after}$$

where $v_{wood\text{-}before} = 0$ m/s; $v_{bullet\text{-}before} = 431$ m/s; $v_{wood\text{-}after} = 1.6446$ m/s; $v_{bullet\text{-}after} = ???$

$$(1.72\ kg) \bullet (0\ m/s) + (0.0085\ kg) \bullet (431\ m/s) = (1.72\ kg) \bullet (1.6446\ m/s) + (0.0085\ kg) \bullet v_{bullet\text{-}after}$$

(To simplify the work, the units will be dropped from the solution in the next several steps. Once a $v_{bullet\text{-}after}$ value is found, its units will be in m/s, consistent with the units stated in the above line.)

$$0 + 3.6635 = 2.8288 + 0.0085 \cdot v_{bullet\text{-}after}$$

$$0.8347 = 0.0085 \cdot v_{bullet\text{-}after}$$

$$(0.8347) / (0.0085) = v_{bullet\text{-}after}$$

$\mathbf{v_{bullet\text{-}after}}$ **= 98.205 m/s = ~98 m/s**

ENERGY LESSON 7 HOMEWORK

DO WORKSHEET (ENERGY IN COLLISIONS)
Comment on the outcomes, paying strict attention to the energy and momentum of each system before and after the collision. **Does the outcome describe a completely inelastic, partially inelastic, partially elastic, completely elastic, or impossible collision?** Provide a brief explanation to accompany each answer.

Energy in Collisions

Type	**total kinetic energy**	**Comments**
perfectly inelastic	decreases to a minimum	objects stick together
Inelastic	decreases by any amount	all collisions between macroscopic bodies
partially elastic or nearly elastic	"nearly conserved"	billiard balls, bowling balls, steel bearings and other objects made from resilient materials
Elastic	absolutely conserved	collisions between atoms, molecules, subatomic particles and other similar microscopic bodies
superelastic	increases	contrived collisions between objects that release potential energy on contact, fictional superelastic materials like flubber

1. Which of the following statements are true about elastic and inelastic collisions?

   a. Perfectly elastic and perfectly inelastic collisions are the two opposite extremes along a continuum; where a particular collision lies along the continuum is dependent upon the amount kinetic energy which is conserved by the two objects.
   b. Most collisions tend to be partially to completely elastic.
   c. Momentum is conserved in an elastic collision but not in an inelastic collision.
   d. The kinetic energy of an object remains constant during an elastic collision.
   e. Elastic collisions occur when the collision force is a non-contact force.
   f. Most collisions are not inelastic because the collision forces cause energy of motion to be transformed into sound, light and thermal energy (to name a few).
   g. A ball is dropped from rest and collides with the ground. The higher that the ball rises upon collision with the ground, the more elastic that the collision is.
   h. A moving air track glider collides with a second stationary glider of identical mass. The first glider loses all of its kinetic energy during the collision as the second glider is set in motion with the same original speed as the first glider.

Since the first glider lost all of its kinetic energy, this is a perfectly inelastic collision.

i. The collision between a tennis ball and a tennis racket tends to be more elastic in nature than a collision between a halfback and linebacker in football

Answer: AEFGI

2. A cricket ball, mass 0.500 kg, was bowled at 50.0 m/s at a batsman who misreads the ball and the 5.00 kg bat is knocked out of his hands, the ball rebounds at 25.0 m/s.

a. What is the initial momentum of the ball?

b. What is the momentum of the ball just after it hit the bat?

c. How much momentum is transferred to the bat?

d. Calculate the velocity of the bat as it leaves the batsman.

e. Calculate the kinetic energies before and after the collision of bat and ball. Is the collision elastic or inelastic?

Answer: 25 kg m/s, -12.5 kg m/s, 37.5 kg m/s, 7.5 m/s, inelastic

3. A ball of mass 0.20 kg is dropped from a height of 3.2 m onto a flat surface which it hits at 8.0 m/s. It rebounds to 1.8 m. ($g = 9.8\ m\ s^{-2}$)

a. What is the rebound speed just after impact?

b. What is the change in energy of the ball?

c. What momentum change has the ball between just touching the surface and leaving it?

Answer: -5.9 m/s, 6.4 J, 2.8 kg m/s

4. At an amusement park, twin brothers Tubby (m = 50 kg) and Chubby (m = 62 kg) occupy separate 36-kg bumper cars. Tubby gets his car cruising at 3.6 m/s and collides head-on with Chubby who is moving the opposite direction at 1.6 m/s. After the collision, Tubby bounces backwards at 0.5 m/s. Assuming an isolated system, determine ...

a. ... Chubby's post-collision speed.

b. ... the percentage of original kinetic energy which is lost as the result of the collision

Answer: (a) v = ~2.0 m/s (b) % KE Loss = ~70%

5. In a physics lab, two carts collide elastically on a level, low-friction track. Cart A has a mass of 1.500 kg and is moving east at 36.5 cm/s. Cart B has a mass of 0.500 kg and is moving West at 42.8 cm/s. Determine the post-collision velocities of the two carts.

Answer: $v_{A\text{-after}}$ = -3.15 cm/s; $v_{B\text{-after}}$ = 76.15 cm/s

6. A classic physics demonstration involves firing a bullet into a block of wood suspended by strings from the ceiling. The height to which the wood rises below its lowest position is mathematically related to the pre-collision speed of the bullet. If a 9.7-gram bullet is fired into the center of a 1.1-kg block of wood and it rises upward a distance of 33 cm, then what was the pre-collision speed of the bullet?

Answer: 2.9×10^2 m/s

7. CHALLEGE QUESTION: You have all the tools to figure this question out. Use what you know and figure out the puzzle.

A 1.72-kg block of soft wood is suspended by two strings from the ceiling. The wood is free to rotate in pendulum-like fashion when a force is exerted upon it. A 8.50-g bullet is fired into the wood. The bullet enters the wood at 431 m/s and exits the opposite side immediately thereafter (i.e. it has exerted a force on the wood giving it a velocity over an instant). If the wood rises to a height of 13.8 cm, then what is the exit speed of the bullet?

Answer: $v_{bullet\text{-}after}$ = ~98 m/s

ENERGY IN COLLISIONS

	before collision	total momentum	total energy
0)	8kg → 6 m/s ; 9 m/s ← 4kg		

	after collision	total momentum	total energy
1)	8 4 (stuck together) → 1 m/s		
2)	1 m/s ← 8kg ; 4kg → 5 m/s		
3)	3 m/s ← 8kg ; 4kg → 9 m/s		
4)	4 m/s ← 8kg ; 4kg → 11 m/s		
5)	6 m/s ← 8kg ; 4kg → 15 m/s		
6)	8 m/s ← 8kg ; 4kg → 5 m/s		

ENERGY IN COLLISIONS

	before collision	total momentum	total energy
0)	8kg → 6 m/s ; 9 m/s ← 4kg		

	after collision	total momentum	total energy
1)	8 4 (stuck together) → 1 m/s		
2)	1 m/s ← 8kg ; 4kg → 5 m/s		
3)	3 m/s ← 8kg ; 4kg → 9 m/s		
4)	4 m/s ← 8kg ; 4kg → 11 m/s		
5)	6 m/s ← 8kg ; 4kg → 15 m/s		
6)	8 m/s ← 8kg ; 4kg → 5 m/s		

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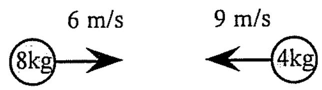
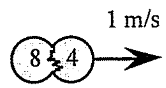
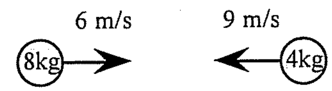
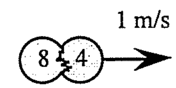
ENERGY IN COLLISIONS

	before collision	total momentum	total energy
0)		+12 kgm/s	306 J

	after collision	total momentum	total energy
1)		+12 kgm/s	6 J
2)		+12 kgm/s	54 J
3)		+12 kgm/s	198 J
4)		+12 kgm/s	306 J
5)		+12 kgm/s	594 J
6)		−44 kgm/s	306 J

ENERGY IN COLLISIONS

	before collision	total momentum	total energy
0)		+12 kgm/s	306 J

	after collision	total momentum	total energy
1)		+12 kgm/s	6 J
2)		+12 kgm/s	54 J
3)		+12 kgm/s	198 J
4)		+12 kgm/s	306 J
5)		+12 kgm/s	594 J
6)		−44 kgm/s	306 J

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