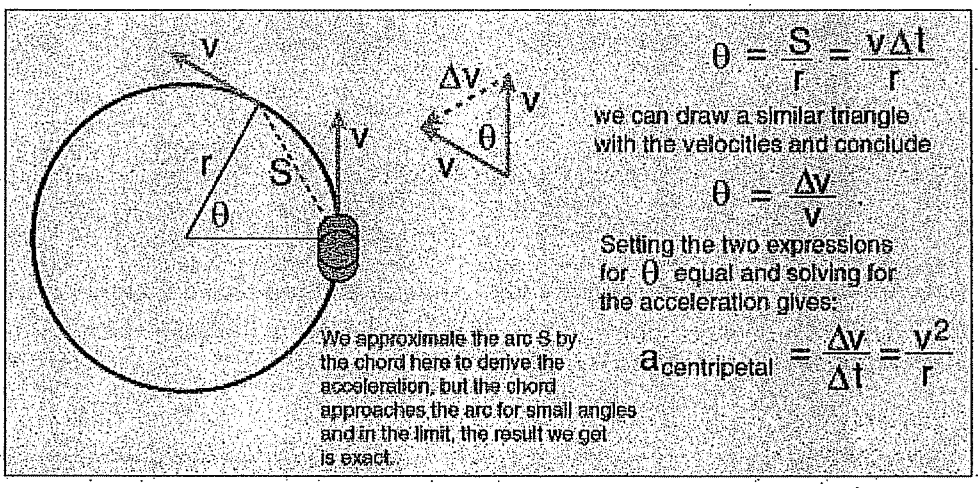
CIRCULAR MOTION

Centripetal Acceleration

For an object with **uniform circular motion**, the **magnitude** of the **velocity remains constant**, but the **direction of the velocity is continuously changing** as it moves around the circle. Its instantaneous velocity at any point around its circular path is in a direction **tangent** to the circular path. A change in direction of velocity constitutes an acceleration just as does a change in magnitude of velocity.

Thus the **centripetal acceleration ("center seeking" acceleration)** of the motion can be derived. Since in radian measure,



An object moving in a circle of radius r with constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is $a_C = \frac{v^2}{r}$

Example: For a velocity of 5.0 m/s and radius 1.0 m, the centripetal acceleration is 25 m/s^2.

Note that if the velocity is doubled to 10.0 m/s at the same radius, the acceleration is **quadrupled** to 100. m/s.

Circular motion is often described in terms of **frequency**, f, as so many revolutions per second. The **period**, T, of an object revolving in a circle is the time required for one complete revolution.

$$T = \frac{1}{f}$$

For example, if an object revolves at a frequency of 3 rev/s, then each revolution takes 1/3 s.

For an object revolving in a circle at **constant speed**, v, the following equation applies:

$$v = \frac{2\pi r}{T}$$

since in one revolution the object travels one circumference ($C = 2\pi r$)

The **centripetal acceleration**:

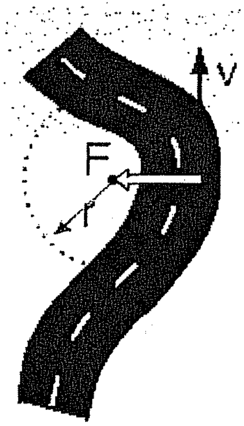
$$a_C = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = \frac{4\pi^2 r}{T^2}$$

Centripetal Force

Any motion in a curved path represents accelerated motion, and requires a force directed toward the center of curvature of the path. This force is called the centripetal force which means "center seeking" force. The force has the magnitude:

$$F_{centripetal} = m\frac{v^2}{r}$$

$\frac{v^2}{r}$ is the centripetal acceleration



The centripetal acceleration can be derived for the case of circular motion since the curved path at any point can be extended to a circle.

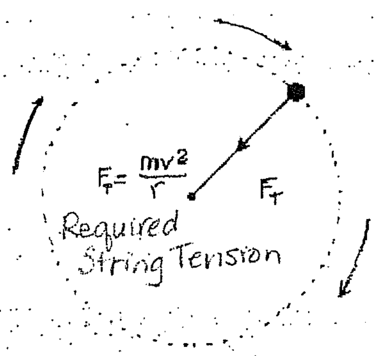
Note that the centripetal force is proportional to the square of the velocity, implying that a doubling of speed will require **four times** the centripetal force to keep the motion in a circle. If the centripetal force must be provided by friction alone on a curve, an increase in speed could lead to an unexpected skid if friction is insufficient.

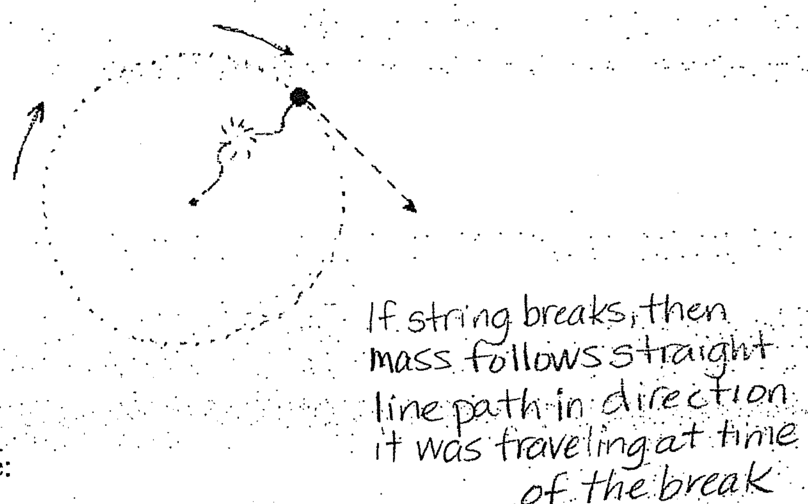
CAN THE CENTRIPETAL FORCE CHANGE THE MAGNITUDE OF THE VELOCITY OF AN OBJECT MOVING IN A CIRCLE?

The answer is NO. The fact that the centripetal force is directed perpendicular to the tangential velocity means that the force can alter the direction of the object's velocity vector but not its magnitude. In order to alter its magnitude, a component of the force would have to be directed in the same or opposite direction of the motion of the object. The centripetal force acts to alter the objects direction into its uniform circular motion, but does not change its velocity magnitude.

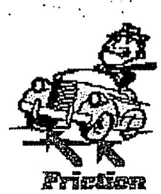
Centripetal Force Example

The string must provide the necessary centripetal force to move the ball in a circle. If the string breaks, the ball will move off in a straight line. The straight line motion in the absence of the constraining force is an example of Newton's first law, ("...objects in motion tend to stay in motion with the same speed and the same direction unless acted upon by an unbalanced force.") The example here presumes that no other forces are acting besides the tension in the string along a horizontal plane. The vertical circle is more involved.

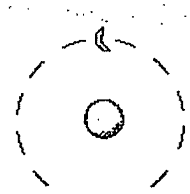




Two other examples of centripetal force are:



As a car makes a turn, the force of friction acting upon the turned wheels of the car provide the centripetal force required for circular motion. If the centripetal force needed to keep the car in the circular path is greater than the force of friction on the wheels, then the car will slide and be unable to make the circular turn.



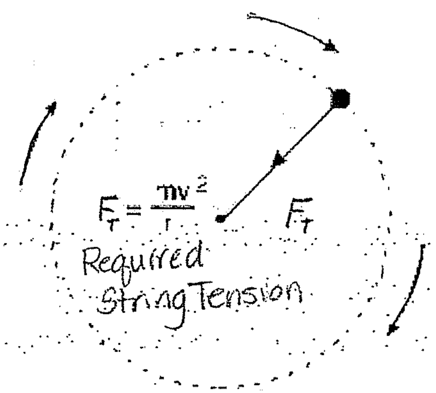
As the moon orbits the Earth, the force of gravity acting upon the moon provides the centripetal force required for circular motion.

Centripetal Force Calculation

$F_C = F_{net} = ma_C$

The idea that $F_{net} = F_C$ will have more meaning next lesson when we solve for vertical circles.

$$F_{net} = m\frac{v^2}{r}$$



Note that the conditions here assume no additional forces, like a horizontal circle on a frictionless surface. F_N balances F_g therefore they do not contribute to F_{net}.

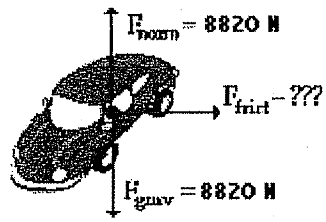
EXAMPLE:

A 900-kg car makes a 180-degree turn with a speed of 10.0 m/s. The radius of the circle through which the car is turning is 25.0 m. Determine the force of friction and the coefficient of friction acting upon the car.

To determine the forces action on the car we first find F_g.

$$\mathbf{F_{grav} = m * g}$$

Knowing that there is no vertical acceleration of the car, it can be concluded that the vertical forces balance each other. Thus, $\mathbf{F_{grav} = F_{norm} = 8820\ N}$. This allows us to determine two of the three forces identified in the free-body diagram. Only the friction force remains unknown.



Since the force of friction is the only horizontal force, it must be equal to the net force acting upon the object. So if the net force can be determined, then the friction force is known. To determine the net force:

$$F_{net} = m * \frac{v^2}{R}$$

$$= 900*10^2/25.0$$

$$= 3600 = \mathbf{3.6 \times 10^3\ N.}$$

Finally the coefficient of friction ("μ") can be determined using the equation which relates the coefficient of friction to the force of friction and the normal force.

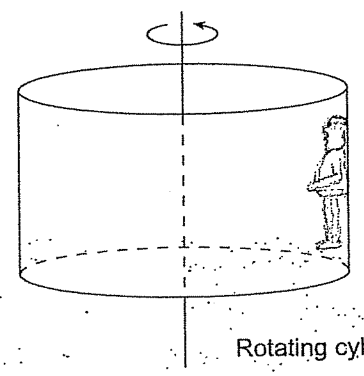
$$F_{frict} = \mu * F_{norm}$$

$$3600 = \mu * 8820$$

$$\mu = 0.408 = \mathbf{0.41}$$

HOMEWORK

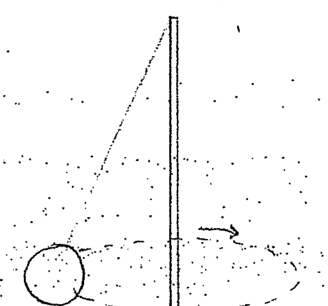
1. Anna Litical is practicing a centripetal force demonstration at home. She fills a bucket with water, ties it to a strong rope, and spins it in a circle. Anna spins the bucket when it is half-full of water and when it is quarter-full of water. In which case is more force required to spin the bucket in a circle? Explain using an equation as a "guide to thinking."
2. The Cajun Cliffhanger at Great America is a ride in which occupants line the perimeter of a cylinder and spin in a circle at a high rate of turning. When the cylinder begins spinning very rapidly, the floor is removed from under the riders' feet. What effect does a doubling in speed have upon the centripetal force? Explain.



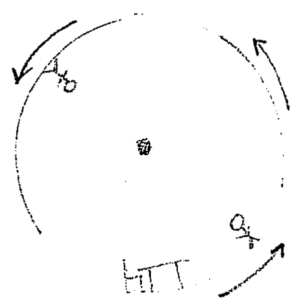
Rotating cylinder

3. Determine the centripetal force acting upon a 40.-kg child who makes 10. revolutions around the Cliffhanger in 29.3 seconds. The radius of the barrel is 2.90 meters. (Ans: 5.3×10^2 N)
4. Rex Things and Doris Locked are out on a date. Rex makes a rapid right-hand turn. Doris begins sliding across the vinyl seat (which Rex had waxed and polished beforehand) and collides with Rex. To break the awkwardness of the situation, Rex and Doris begin discussing the physics of the motion which was just experienced. Rex suggests that objects which move in a circle experience an outward force. Thus, as the turn was made, Doris experienced an outward force which pushed her towards Rex. Doris disagrees, arguing that objects which move in a circle experience an inward force. In this case, according to Doris, Rex traveled in a circle due to the force of his door pushing him inward. Doris did not travel in a circle since their was no force pushing her inward; she merely continued in a straight line until she collided with Rex. Who is correct? Argue one of these two positions.
5. Kara Lott is practicing winter driving in the KSS parking lot. Kara turns the wheel to make a left-hand turn but her car continues in a straight line across the ice. Teacher A and Teacher B had viewed the phenomenon. Teacher A argues that the lack of a frictional force between the tires and the ice results in a balance of forces which keeps the car traveling in a straight line. Teacher B argues that the ice placed an outward force on the tire to balance the turning force and thus keep the car traveling in a straight line. Which teacher is (A or B) is the physics teacher? ______ Explain the fallacy in the other teacher's argument.

6. A 900. kg car moving at 10. m/s takes a turn around a circle with a radius of 25.0 m. Determine the acceleration and the net force acting upon the car? (Ans: 4.0 m/s^2, 3.6×10^3 N)
7. A 95 kg halfback makes a turn on the football field. The halfback sweeps out a path which is a portion f a circle with a radius of 12 meters. The halfback makes a quarter of a turn around the circle in 2.1 seconds. Determine the speed, acceleration and net force acting upon the halfback. (Ans: 9.0 m/s, 6.7 m/s^2, 6.4×10^2 N)
8. A 150. g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions in a second. What is the centripetal acceleration? (Assume the only force on the ball is the centripetal force) (Ans: 94.8 m/s^2)
9. A ball of mass 0.50 kg is attached to the end of a cord whose length is 1.5 m. The ball is whirled in a horizontal circle. If the cord can withstand a maximum tension of 50. N, what is the maximum speed that ball can have before the cord breaks? (Assume the mass small enough that gravity is negligible)
10. The coefficient of friction acting upon a 900.-kg car is 0.850. The car is making a 180-degree turn around a curve with a radius of 35.0 m. Determine the maximum speed with which the car can make the turn. (Ans:17.1 m/s)
11. The game of tetherball is played with a ball tied to a pole with a string. When the ball is struck, it whirls around the pole as shown in the diagram. In what direction is the acceleration, and what causes the acceleration? If the tetherball is 2.5 kg and the radius of its circular path is 0.95 m while suspended from a string of length 1.5 m at 32° from pole, what is the centripetal force on the tetherball? What is the velocity (magnitude) of the tetherball? (Ans: 15 N, 2.4 m/s)



12. A device for training astronauts and jet fighter pilots is designed to rotate the trainee in a horizontal circle of radius 10.0 m. If the force felt by the trainee is 7.75 timers her own weight, what is her period of rotation? How fast is she rotating? (Ans: 2.3 s = 1 revolution takes 2.3 seconds, 28 m/s)
13. Astronauts who spend long periods in outer space could be adversely affected by weightlessness. One way to simulate gravity is to shape the spaceship like a cylindrical shell that rotates, with the astronauts walking on the inside surface.



(b) $a_C = \dfrac{4\pi^2 r}{T^2}$

$\frac{1}{2} * 9.8 = \dfrac{4\pi^2 32}{T^2}$

$T = \sqrt{\dfrac{4 * \pi^2 32}{0.5 * 9.8}}$ = 16.0485 = **16 s = 1 revolution takes 16 seconds**

(c) The period is decreased and therefore the centripetal force increase $F_C \propto \dfrac{1}{T^2}$.
Since the centripetal force is only provided by the normal force, **the normal force on the astronaut increases (F_N is perceived as weight) The astronaut feels heavier.**

(a)Explain how this simulates gravity. Consider (a) how objects fall, (b) the force we feel on our feet, and (c) any other aspects of gravity you can think of.

(b) At what rate must the cylindrical spaceship rotate, if occupants are to experience simulated gravity of ½ g? Assume the spaceship's diameter is 32 m and give your answer as the time needed for one revolution. , i.e. the Period. (Ans: 16 s, one revolution would take 16 seconds)

(c)What would be the effect experience by the astronauts if the space station rotated faster so that the period of rotation was decreased? Explain your predicted effect.

ANSWERS –

1. It will require more force to accelerate a full bucket of water compared to a half-full bucket. According to the equation, $F_C = mv^2/r$, force and mass are directly proportional, so the greater the mass, the greater the force.
2. Doubling the speed of the ride will cause the force to be 4x greater than the original force. According to the equation, $F_C = mv^2/r$, force and v^2 are directly proportional. So 2x the speed means 4x the force (that's from 2^2)
3. First find the speed,
   $v = 2\pi r/T$,
   T = 2.93 s, since 10 cycles takes 29.3.
   Therefore, v = 6.22 m/s.

   Then find acceleration, $a = v^2/r = 13.3\ m/s^2$

   Now find force, $F = ma = 5.3 \times 10^2$ N

   Could also use the equations: $a_C = \frac{4\pi^2 r}{T^2}$ and $F = ma_C$

4. Doris is correct. When the turn is made, Doris continues in a straight line path; this is Newton's 1st Law of Motion. (Objects in motion tend to stay in motion with the same speed and the same direction unless acted upon by an unbalanced force) Once Doris collides with Rex, there is an unbalanced force capable of accelerating Doris towards the center of the circle, causing the circular motion.
5. Teacher A is correct. A car turns in a circle due to friction against its turned wheels. With wheels turned and no friction, there would be no circle. That is the problem in this situation.
6. To determine the acceleration of the car, use the equation $a = (v^2)/R$. The solution is as follows;

$$a = (v^2)/R$$

$$a = ((10.0\ m/s)^2)/(25.0\ m)$$

$$a = (100\ m^2/s^2)/(25.0\ m)$$

$$\mathbf{a = 4.0\ m/s^2}$$

To determine the net force acting upon the car, use the equation Fnet = m*a. The solution is as follows.

$$F_{net} = m*a$$

$$F_{net} = (900\ kg)*(4\ m/s^2)$$

$$\mathbf{F_{net} = 3.6 \times 10^3\ N}$$

7. To determine the speed of the halfback, use the equation v = d/t where the d is one-fourth of the circumference and the time is 2.1 s. The solution is as follows:

$$v = d/t$$

$$v = (0.25 * 2 * pi * R)/t$$

$$v = (0.25 * 2 * 3.14 * 12.0 \text{ m})/(2.1 \text{ s})$$

$$\mathbf{v = 8.97 = 9.0 \text{ m/s}}$$

To determine the acceleration of the halfback, use the equation $a = (v^2)/R$. The solution is as follows:

$$a = (v^2)/R$$

$$a = ((8.97 \text{ m/s})^2)/(12.0 \text{ m})$$

$$a = (80.5 \text{ m}^2/\text{s}^2)/(12.0 \text{ m})$$

$$\mathbf{a = 6.7 \text{ m/s}^2}$$

To determine the net force acting upon the halfback, use the equation Fnet = m*a. The solution is as follows.

$$F_{net} = m*a$$

$$F_{net} = (95.0 \text{ kg})*(6.71 \text{ m/s}^2)$$

$$\mathbf{F_{net} = 637 = 6.4 \times 10^2 \text{ N}}$$

8. The centripetal acceleration is $a_C = v^2/r$

First we determine the speed of the ball, v.
If the ball makes two complete revolutions per second, then the ball travels in a complete circle in 0.500 s, which is its period, T. The distance traveled in this time is the circumference of the circle, $2\pi r$, where r is the radius of the circle. Therefore the ball has speed,

$$v = \frac{2\pi r}{T} = \frac{2*3.14*0.600}{0.500} = 7.536 \text{ m/s}$$

The centripetal acceleration is:

$$a_C = \frac{v^2}{r} = \frac{7.536}{0.600} = 94.652 = \mathbf{94.7 \text{ m/s}^2}$$

9. Because the centripetal force in this case is the tension T in the cord, we get:

$$F_{net} = F_{tension} = \frac{mv^2}{r}$$

The maximum speed that the ball can have will correspond to the maximum value of the tension, therefore:

$$V_{max} = \sqrt{\frac{F_{tension\,max} * r}{m}} = \sqrt{\frac{50*1.5}{0.50}} = 12.2474 = \mathbf{12\ m/s}$$

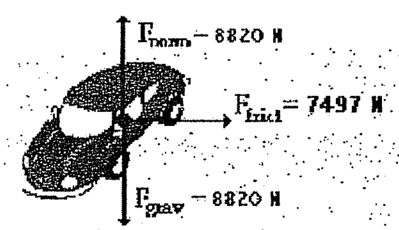
10. The normal force, F_N, is equal to the weight since the road is flat, and there is o vertical acceleration:

$$\mathbf{F_{grav} = F_N = m * g}$$
$$= 900*9.8 = 8820\ \text{N}$$

Since the coefficient of friction ("μ") is given, the force of friction can be determined using the following equation:

$$F_{frict} = \mu * F_{norm}$$
$$= 0.850*900*9.8$$
$$= 7497\ \text{N}$$

This allows us to determine all three forces identified in the free-body diagram.



The net force acting upon any object is the vector sum of all individual forces acting upon that object. So if all individual force values are known (as is the case here), the net force can be calculated. The vertical forces add to 0 N. Since the force of friction is the only horizontal force, it must be equal to the net force acting upon the object. Thus, $\mathbf{F_{net} = 7497\ N}$.

Once the net force is determined, the acceleration can be quickly calculated using the following equation:

$$\mathbf{F_{net} = m*a}$$
$$7497 = 9.8*a$$

$= \underline{8.33 \text{ m/s}^2}$

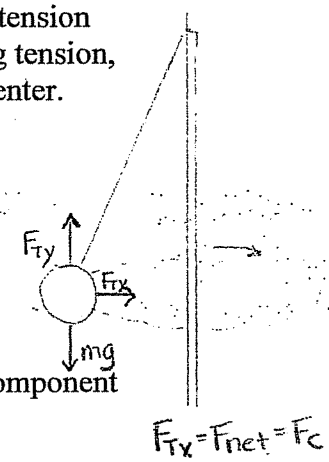
Finally, the speed at which the car could travel around the turn can be calculated using the equation for centripetal acceleration:

$$a = \frac{v^2}{R}$$

$$8.33 = v^2/35.0$$

$$v = 17.07 = \mathbf{17.1 \text{ m/s}}$$

11. The acceleration point horizontally toward the center of the ball's circular path (not toward the top of the pole). The force responsible for the acceleration may not be obvious at first, since there seems to ve no force pointing directly horizontally. But is it the net force (the sum of mg and F_T here) that must point in the direction of the acceleration. The vertical component of the string tension balances the ball's weight, mg. The horizontal component of the string tension, F_{Tx}, is the force that produces the centripetal acceleration toward the center.



To determine the centripetal force we must determine the horizontal component of the tension. We can do this by using trig:

$$\tan \theta = \frac{F_{net}}{F_g}$$

$$\tan 32 = \frac{F_{net}}{2.5 * 9.8}$$

$F_{net} = 15.309 = \mathbf{15\ N}$

To determine the velocity of the tetherball, we use the following equation:

$$F_{net} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{F_C * r}{m}} = \sqrt{\frac{15.309 * 0.95}{2.5}} = 2.4119 = \mathbf{2.4 \text{ m/s}}$$

12. $\mathbf{F_{net} = m*a_C}$

$$\mathbf{F_{net}} = \frac{m*4\pi^2 r}{T^2}$$

$$7.75*m*9.8 = \frac{m*4\pi^2 10.0}{T^2}$$

m's cancel out and we are left with:

$$T = \sqrt{\frac{4*\pi^2 10.0}{7.75*9.8}} = 2.2787 = \mathbf{2.3\ s} = \textbf{ 1 revolution takes 2.3 seconds}$$

To find the velocity, we use the following equation:

$$F_C = \frac{mv^2}{r}$$

$$7.75*m*9.8 = \frac{mv^2}{10.0}$$

masses cancel out and we have:

$$v = \sqrt{7.75*9.8*10.0} = 27.559 = \mathbf{28\ m/s}$$

13. (a) In space, no gravitational forces act on objects (not entirely true since there is slight gravitational pulls due to planets in space, but we will assume they are negligible), therefore the astronauts are stationary unless they push or pull on something to create a force on themselves and therefore cause movement and they are weightless (no normal force acts against their feet). If the cylinder where to rotate, the astronauts would have a constant centripetal force applied to their feet which would act as their normal force, i.e. the force exerted on them by the floor. This gives a feeling of weight. What happens if they are to jump up from the floor, would they come back down to the floor like on earth. Yes, think about a spinning cylinder. They would have a vertical velocity but they would also have a horizontal velocity, i.e. the same speed as the rotation of the cylinder, so in effect they would land back down on the floor of the cylinder where they left it. See Diagram